Foundations (Part 1.A) - Understanding Bode Plots and Stability of Power Supplies

tags: bode plot, stability, crossover frequency, phase margin, gain margin

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Introduction

Most experts agree that the best way of designing power supply control loops is through the use of frequency domain design methods. Some people may have studied these techniques at university in their "Control Theory" lectures but perhaps not applied directly to power supplies.

Please don't let the phrase "Control Theory" put you off reading the rest of this article; we have come up with a way of making it not just tolerable, but perhaps even easy and fun.

Control theory was perhaps one of the subjects that I disliked most at university (second only to Maxwell's Equations!), both as a student and as a lecturer. The reason was that it was never related to real life.

In this article we will relate everything to real power supplies. By the end of the article you should be very comfortable with the concepts of frequency domain analysis, Bode plots, gain margin, phase margin, cross over frequency and finally the power supply stability criteria. But let us first explain how and why understanding these topics are important for designing stable real life power supplies.

Frequency Domain Design

What is frequency domain design and why is it better than the time domain when applied to power supplies?

As mentioned earlier, most experts design power supplies in the frequency domain, the main reason is that when we work in the frequency domain, we obtain a lot more information about the relative stability of our power supplies. In the time domain (i.e. when we give the power supply a step load and look at its transient response) we can never be completely certain of how far away from instability we are.

By frequency domain analysis, what I mean is that I inject a small sinusoid of a certain frequency, let's say 10 Hz, into my power supply and then measure how the shape of this sinusoid is modified by the time it goes through the PSU and eventually comes out. This will give me the stability information that I need in order to design a nice compensator.

But how I do I inject this sine wave into my power supply? Well, assuming for now that I only have a simple proportional controller with a gain of 1, I get my PWM signal and I give its pulse-width a small wiggle/shake at for example 10Hz. This injects a 10Hz sinusoid into my power supply or my "plant". Then I wait for this sine wave to go all the way around the system and I look at the sinusoid that comes out of the plant. By the way, in next month's article we will go through the exact real life mechanism by which this is done in a real power supply.

Having injected a sine wave and looking at its shape as it comes out of my power supply, I can then compare the sine wave that I put in, with the sine wave that I have got out. When we compare these two sine waves, there are only two things that can really change.

One is the height difference between the two sine waves and that gives us our "gain" and the other one is the "phase" difference between them.

For example if we injected a 1V sine wave into our system and we saw a 10V sine wave on the output then our gain at 10 Hz would be:

$$\text{Gain} = \left(\frac{10}{1}\right) \Rightarrow 20\log(10) = 20\text{dB}$$

Equation 1

And if at 10Hz, the sine wave that came out lagged the sine wave that I injected by 90^{o} , then our phase at 10Hz would be -90^{o}.

I can now put two points on my Bode plot at 10Hz; one on my gain plot and one on my phase plot. Then I repeat the process for 20Hz, 30Hz, 40Hz, 50Hz and I plot these gains and phases all the way up to half the switching frequency.

The gain and phase plots obtained are that of the open loop Bode plot of the power supply before compensation (remember that we assumed we have a simple proportional controller with a gain of 1). Figure 1 shows an arbitrary Bode plot of a voltage mode power supply including a compensator. We will explain what to look for in a Bode plot next and discuss compensator design in later articles. For now let us discuss what our Bode plot should look like in order to achieve my design goals.

Interpreting Bode Plots

Interpreting Bode plots for stability is in fact very easy. We are mainly only looking for 4 things:

1. Cross over frequency, Fx
2. Phase Margin, Pm
3. Gain Margin, Gm
4. Slope of the gain plot at the cross over frequency

Just these 4 attributes will tell us almost everything that we need to know about the stability of our power supply. Let us discuss what they are and why they are important.

Cross over frequency

Cross over frequency Fx: Cross over frequency, is a measure of how quickly our power supply recovers from a step change in the load. The higher our cross over frequency, the faster our step response in time domain. Also, we need to know the cross over frequency in order to ascertain our phase margin (more on this shortly).

The cross over frequency is defined as the frequency at which the gain plot crosses the 0dB axis. But what does this actually mean? Remember that we are injecting sine waves of different frequencies and measuring the gain of these which we plot in dB.

If we injected a 1V sine wave into our system and the sine wave that came out was exactly the same height we would have a gain of 1 which, in dB world, is a 0dB:

$$20log\left(1\right) = 0dB$$

Equation 2

From the above you can see that when the height of the sine wave that we inject into our system is the same as the height of the sine wave that comes out, our gain is 0dB and hence our gain plot will cross the zero dB axis. The frequency at which this happens is called the cross over frequency.

Consider the Bode plot of a simple Buck converter shown in Figure 1. Look at the gain plot and you will see that at around 11kHz our gain becomes 0dB i.e. our cross over frequency for this particular power supply is 11kHz.

For the power supply shown the switching frequency is 200kHz. For stable robust operation our cross over frequency should be no higher than 1/10th of our switching frequency.

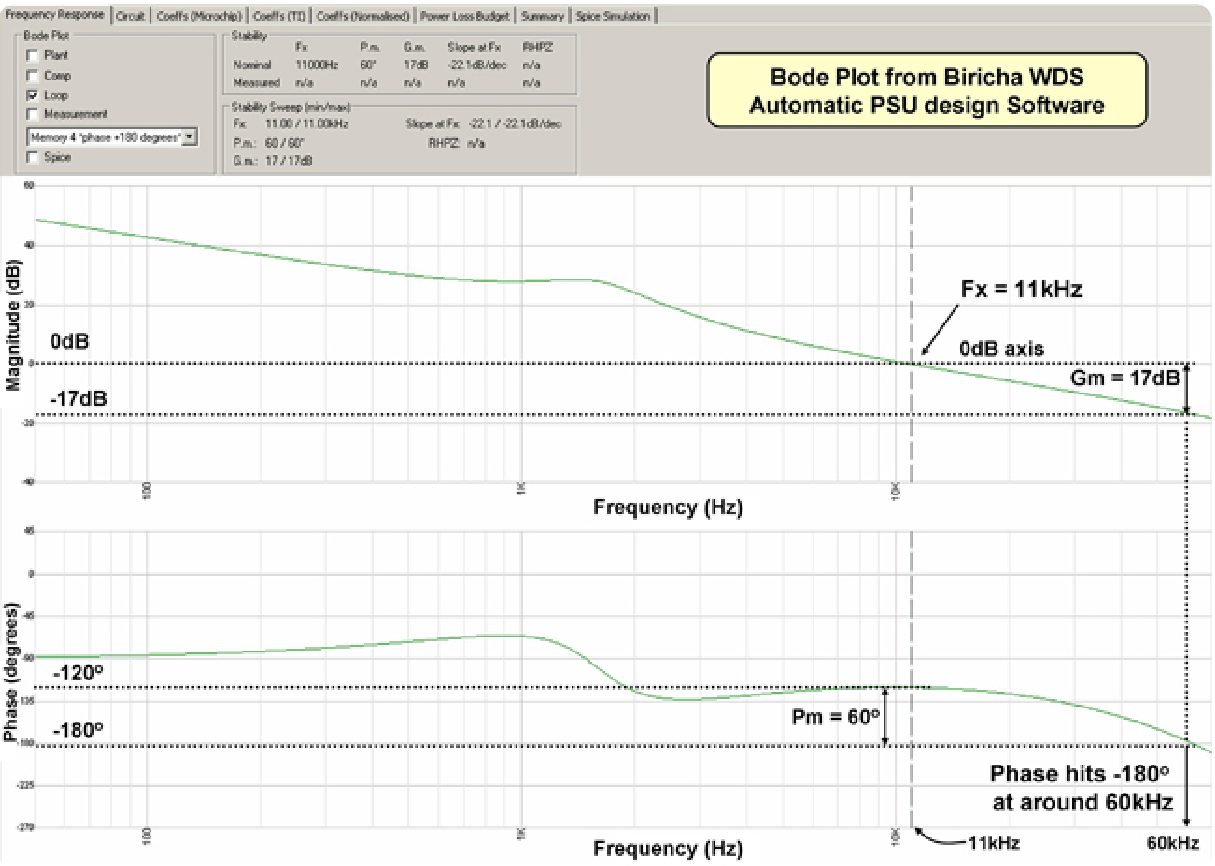


Figure 1. Simulated Bode plot of a classic voltage mode Buck converter. Images taken from Biricha Digital's Automatic PSU Design Software (Biricha WDS)

Phase Margin

Phase Margin, Pm: Now that we have defined cross over frequency, we can define our phase margin which is perhaps the most important term you need to know in order to ensure stability. Phase margin is a measure of the relative stability of our power supply.

Our phase margin is defined as how much the phase lag is above -180° at the cross over frequency. Looking at our Bode plot, our cross over frequency was 11kHz. Please take a look at our phase at 11kHz. You will see that our phase is about -120°. The minus sine means that the sine wave that came out had a phase lag of 120º compared to the sign wave that we injected. How much is our phase above -180º ? Well it is 180º - 120º = 60º.

As our phase margin reduces our system behaviour becomes more and more oscillatory. If our phase margin ever becomes 0º, then we would have a perfect oscillator, which in engineering world we would call unstable. Therefore to be on the safe side we would like to have a certain amount of "safety margin" with our phase before hitting oscillations; this is called our phase margin and we should aim for at least 45º under worse case conditions. We recommend designing for 55 to 60 degrees to be on the safe side.

Gain Margin

Gain Margin Gm: Similar to phase margin, our gain margin is also a measure of our relative stability. If you look at our phase plot, you will see that at some frequency, the phase crosses -180º. If you look at your gain at that exact frequency you have ascertained your gain margin. In our case the phase hits -180º at around 60kHz. At 60kHz our gain is around 17dB below 0dB. This means that our gain margin is 17dB.

Again as gain margin approaches 0dB then our power supply will oscillate. So for a robust power supply we would like to have about 10dB of gain margin; i.e. our gain needs to be 10dB below 0dB when the phase hits -180º.

Slope of the Gain Plot at the Cross Over Frequency: The forth and final piece of the stability jigsaw can be ascertained by looking at the slope of the gain plot at cross over. For a robust and stable system ideally you would like to cross over with a shallow slope of around -20 to -25 dB per decade. Here, by "decade" we mean our frequency changing by a factor of 10. For example if our gain at a 5kHz is 20dB then at a frequency 10 times higher (i.e. 50kHz) gain should not have fallen by more than 20dB. So the gradient of the gain plot at cross over needs to be shallow rather than sharp. A sharp slope at cross over frequency could bring us dangerously close to instability. From our Bode plot we can see that our slope is about -22dB per decade.

Real Life Example

To put this theory into practice we will now look at a Bode plot showing real measurement data from a voltage mode controlled Buck converter. The PSU has been measured using the Bode 100 frequency response analyzer from OMICRON Lab. We will discuss the details of how the measurement was taken in the next article, however, for now we will just assess the stability of this real power supply.

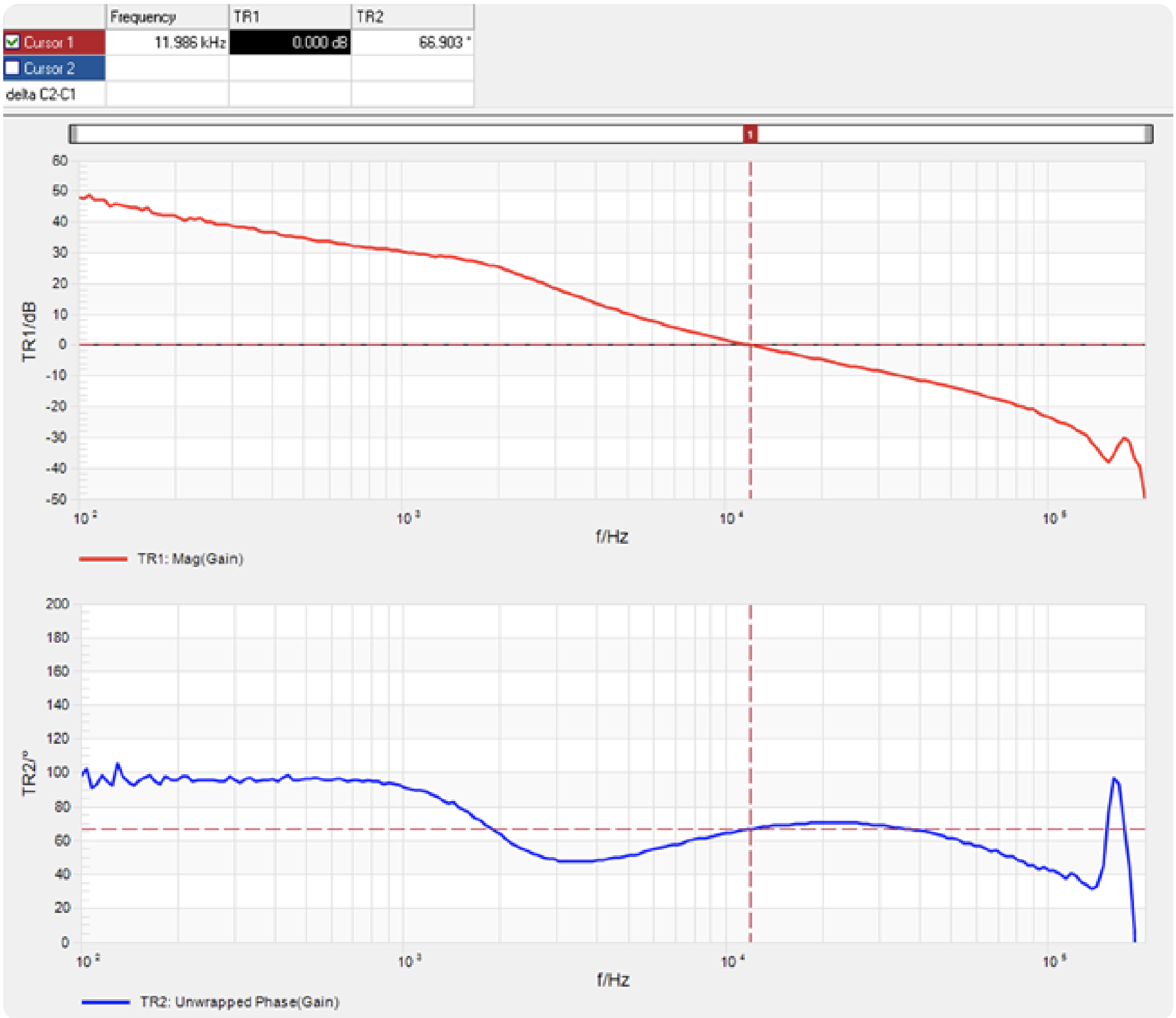


Figure 2. Measured Bode plot of a voltage mode Buck converter. Measurement taken using Bode 100 Vector Network Analyzer from OMICRON Lab

The Bode plot shown in Figure 2 has a cross over frequency of around 12kHz. As we now know, this is the frequency at which the gain plot crosses the 0dB axis. The switching frequency for this power supply is 200kHz and therefore our cross over frequency is less than 1/10th of our switching frequency - meeting the first condition that we outlined. The phase margin at this frequency can be measured as 67° using the cursors within our network analyzer's software. This is greater than the 45° minimum and therefore the power supply meets our second objective. Please note that on this particular plot we start counting from 0° as opposed to -180°. This is due to the inverting action of our power supply's op-amp. This simply means that our phase margin is displayed as how many degrees above 0° we are as opposed to -180°. Apart from this simple change of reference point there is no other difference.

Now let us determine our gain margin. You can see that our phase in fact does not hit -180° (or 0° in this plot) as we earlier defined it. However we can see that the gain margin is already better than 20dB by the time we have reached half the switching frequency. This is great and clearly our gain margin will be better than our 10dB minimum even though we can not see it.

Finally, the slope at cross over is shallow, around -22dB per decade, and thus we meet the final condition that we have outlined. Having met all 4 conditions set out earlier in the stability criteria, we can conclude that this is a stable, well tuned power supply.

Concluding Remarks

In this article we have explained the foundations of frequency domain stability analysis. We have explained how Bode plots are created and what to look for on a Bode plot for a stable power supply. In short, in order to have a stable power supply, we should meet the following conditions:

1. Cross over frequency, i.e. the frequency at which the gain plot crosses the 0dB axis should not be higher than 1/10th of our switching frequency
2. Phase margin, i.e. how much our phase is above -180° at cross over frequency, should not be less than 45°
3. Gain Margin, i.e. how much our gain is below 0dB when our phase hits -180° should not be less than 10dB
4. Slope of our gain plot at cross over, i.e. the gradient of our gain plot should be shallow and as close to -20dB per decade as possible

If we achieve the above four conditions, then we will have a stable power supply. Now that we know what to look for in a Bode plot, in the next article we will discuss in detail exactly how we measure the frequency response of our power supply in real life. We will explain how to measure and plot the Bode plot of the plant i.e. the power stage, the compensator and of course the entire loop.

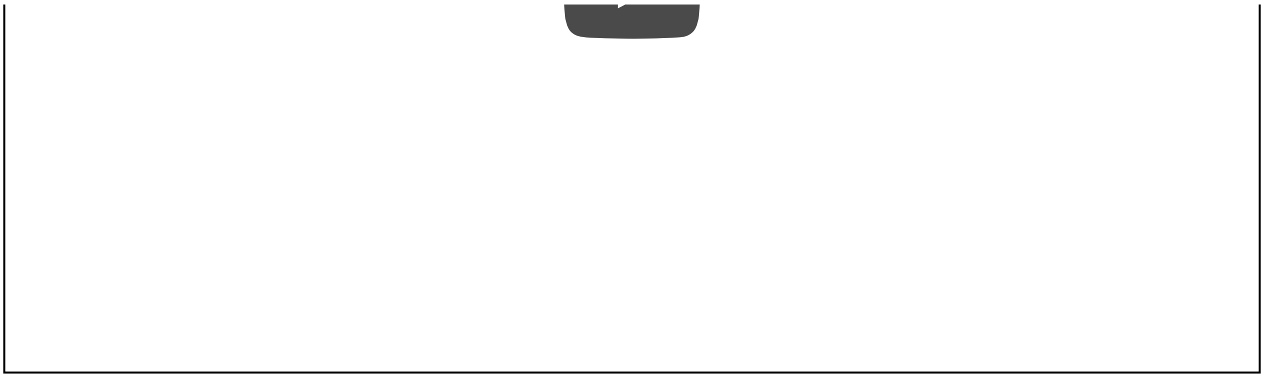
Things to Try

Check out our video explaining Bode plots in less than 10 minutes below

Foundations (Part 1.A) - Understanding Bode Plots and Stability ...

Dr. Ali Shirsavar from Biricha Digital Power Ltd explains how to analyze a Bode plot of a switched-mode power supply. The terms crossover frequency, phase margin and gain margin are defined and a set of stability criteria are outlined for robust and stable power supplies.

Foundations (Part 1.A) - Understanding Bode Plots and Stability ...



Dr. Ali Shirsavar from Biricha Digital Power Ltd describes Bode plots in terms on stability of a power supply and measures the loop of a Buck converter using a Bode 100 vector network analyzer from Omicron Lab.

Please visit our store for more information and to purchase an Omicron Lab Bode 100 Vector Network Analyzer (/store)

You can download an evaluation version of our WDS PSU Design software and experiment with Bode plots here (/wds).

Bibliography

[1] Biricha Digital's Analog Power Supply Design Workshop Manual

[2] Switching Power Supplies A-Z, by Sanjaya Maniktala, ISBN: 0123865336

[3] Switch-Mode Power Supplies : SPICE Simulations and Practical Designs, by Christophe Basso, ISBN: 8183332919